Practice Problem Set 3

Question 1 A professor seeks to evaluate his students in a class in economics. He knows that 30% of the students are weak, 50% are average and 20% are good. The professor's objective is to distinguish between good and bad students. We suppose that he receives a payoff of $+1$ if a good student passes the exam, of $+1$ if a bad student fails the exam, of -1 if a good student fails the exam, of -1 if a bad student passes the exam, and of 0 for any average student, irrespective of their performance at the exam.

(a) The professor first drafts a very easy exam that 80% of the weak students, 90% of the average students and 100% of the good students will pass. What are the probabilities that a student who passes the exam is weak/average/good? What is the expected payoff to the professor?

(b) The professor then drafts a very hard exam that 0% of the weak students, 10% of the average students and 20% of the good students will pass. What are the probabilities that a student who passes the exam is weak/average/good? What is the expected payoff to the professor?

(c) Finally, the professor drafts an average exam that 40% of the weak students, 50% of the average students and 60% of the good students will pass. What are the probabilities that a student who passes the exam is weak/average/good? What is the expected payoff to the professor?

(d) What is the optimal choice of the professor?

Question 2 We consider an exchange economy with two consumers 1 and 2, two goods x and y and two states of nature α and β. The two states of nature have the same objective probability $Pr(\alpha) = Pr(\beta) = \frac{1}{2}$. Consumers have a utility function which is independent of the state of nature:

$$U(x, y) = \sqrt{xy}.$$

In state α, consumers have endowments $e_1(\alpha) = (2, 0), e_2(\alpha) = (0, 1)$ and in the other state β, endowments $e_1(\beta) = (1, 0), e_2(\beta) = (0, 2)$. The price of good x is normalized to 1 and the price of good y is denoted p. Consumer 1 knows the state of nature but not consumer 2.

(a) Suppose that the Rational Expectations equilibrium is revealing, $p(\alpha) \neq p(\beta)$. Compute the demands of consumers 1 and 2 in the two states of nature for these prices.

(b) Compute the equilibrium prices $p(\alpha)$ and $p(\beta)$ in the Rational Expectations equilibrium of this exchange economy.